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THE MATHEMATICS TEACHER

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AN EXPERIMENT IN MOTIVATION.

BY WILLIAM S. SCHLAUCH.

Educators are generally agreed that the genesis of knowledge in the individual should follow the line of evolution by which the race conquered and learned to interpret its environment. When challenged by a new situation man bent his energies to the solution of the problem that perplexed him. His environment presented a series of problems to him and out of the series of efforts to feed, clothe, and shelter himself, and harness the forces of Nature the sciences gradually differentiated out of a mass of common knowledge. Mathematical relations were investigated first because they were involved in solving some definite problem in which the investigators were interested.

The usual procedure in our high schools, and even in the elementary schools, is to present differentiated and organized sciences to our students, prescribing algebra, geometry, and trigonometry as "requirements for graduation," taking little thought of whether or not the student is interested in acquiring a working knowledge of quadratic equations, series, sines, cosines and tangents. The trite observation that students learn a subject much faster if they are interested in it than if it is presented to them as a task to be learned "willy-nilly," has so far born little fruit in the shape of presenting mathematics or sciences as a way of exploring the world or solving problems in which the student is really and immediately interested.

During the summer session of 1918 at New York University

the author had a number of interesting discussions with Prof. James E. Lough, director of the summer school, and dean of the extra-mural division of the university, about methods of presenting mathematics to students in such fashion that it would be a tool for investigating and solving problems in which the student's interest is already aroused. As professor of psychology he was eager to try the experiment of presenting trigonometry and even algebra as collateral and necessary tools of investigation in the fields of artillery practice and navigation. We agreed to offer two courses in the extra-mural division, beginning late in September, 1918. One called "The Mathematics of Navigation," and the other "The Mathematics of Gunnery." No college credit was to be allowed for either course. They were opened to students who had an elementary knowledge of algebra. The dean generously allowed students of the High School of Commerce to enter the courses free of charge. They were given in the afternoon, from 3:30 to 5:30 P.M., each once a week. Only students who expected to be drafted and looked forward to entering the artillery or naval arms of the service were appealed to. Little effort to advertise the courses outside the High School of Commerce was made, and the courses were made up almost exclusively of Commerce boys in the senior and junior classes. We expected a half dozen students for each course. Twenty registered for the course in artillery, and nineteen for the course in navigation.

Bear in mind that all these students with three exceptions had a full day's program of study and recitation as students in the High School of Commerce; that they were willing to put in two hours a week in addition to their regular school work, studying a subject for which they would receive neither credit toward graduation, nor credit at the university. They wanted the knowledge offered in the courses to help them in solving a real problem—that of advancing more rapidly in the service if called by the government to serve in the war. The motive was real and compelling. Day after day these young men met and learned as much trigonometry in one lesson as is covered by the ordinary student in a week or more. No teacher ever had more absorbed attention, or more determined spirit on the part of his students.

And note that the students were not told that they were being taught physics, algebra and trigonometry as collaterals of the courses. They were studying navigation and gunnery calculations, and that was the center and focus of attention, and they were interested in those subjects because they expected to enter those fields. They visualized a situation as impending that would bring these fields into their lives, and the response in interest was remarkable. Up to the time the armistice was signed the classes remained full. About half dropped out the week after the armistice was signed. Those who remained had become so much interested in the subject matter itself that they remained because of the pleasure they derived "from successful functioning."

The method of introducing the mathematics as a tool for investigating the problems of artillery or navigation may be best understood from specific examples. After presenting some information about the three-inch field piece, explosives, internal pressure and rifling, the student studied the path of a trajectory in vacuo, drawing or plotting to scale on cross section paper the path of the trajectory having a given initial velocity, the angle of elevation being also given. The path was plotted by combining the factors

$$S = vt$$

and

$$S' = \tfrac{1}{2}gt^2.$$

The position of the trajectory for $t = 1, 2, 3, \ldots$ was plotted and the curve thus located. Then the range table for the trajectory in air given in War Department Document No. 391 was taken up. The student was made familiar with the concepts of range, angle of departure, quadrant angle of departure, the jump, drift, angle of fall, etc. The experimental methods involved in making the table were described. The whole situation in the field, with wind, air pressure, and other modifying circumstances was made a reality for the students. The columns of the table then had a meaning for the students. But the impossibility of always having a range table at hand occurred to a number of students, who introduced the question "Can't you tell the range K, when you know ϕ in mils without a table?"

The lecturer followed this lead and presented the solution of the problem raised in what would have been called in a formal lecture course "Rapid Calculation of the Elements of the Trajectory." And the solution of this problem, which involved the empirical equation

$$\phi \text{ (in mils)} = 5K(K+3)$$

led to enough practice and comparison with the figures of the table, to familiarize the students with the equation, so that when the range K was given in thousands of yards, the angle of departure ϕ in mils was easily and rapidly found by the class. The discussion brought out the question: Suppose your angle of departure is set, can you tell by an easy calculation how far your shell will go? The students themselves by a leading question discovered that they would have to solve

$$\phi = 5K(K+3)$$

for K. Now, practically all of the class had never had quadratic equations. They agreed that it would be much better to learn how to solve one equation for a second letter than to try to memorize two equations. So they were given a lesson on completing the square and solving quadratics, as well as a short drill on simplifying radicals. The resulting equation

$$K = -\frac{3}{2} + \sqrt{\frac{\phi}{5} + \frac{9}{4}}$$

or

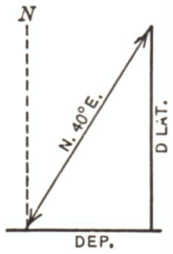
$$K = \sqrt{\frac{\phi + 11}{5}} - 1.5$$

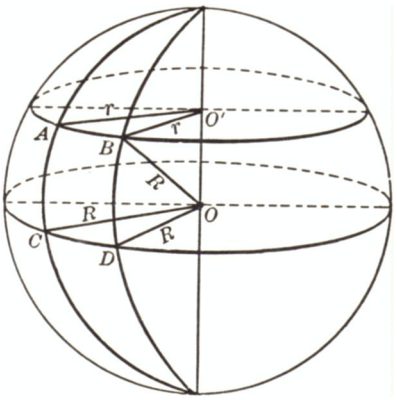
was more easily memorized because of the attention and thought processes involved in deriving it. Besides, they did not need to memorize it. They could derive it at any time. To those students, quadratic equations mean a tool for working out a problem in which they were interested.

Again, consider the mathematical avenues opened by a consideration of the problem of firing from behind a mask.

In the diagram, G is the gun, T the target, y is the mask, t is the actual trajectory and P is the Percin parabola, whose ordinates are always smaller than those of the actual trajectory of

the same range, R. The problem is to find x, the distance behind the mask whose height is y yards, that the gun must be placed so that the shell will clear the mask.





Percin's parabola has the equation

$$4y = x(R - x),$$

in which R = entire range from gun to target in *hundreds of yards*, y is the ordinate in yards corresponding to any abscissa x expressed in hundreds of yards. $R - x$ and y are known by range finder and measurements. Then

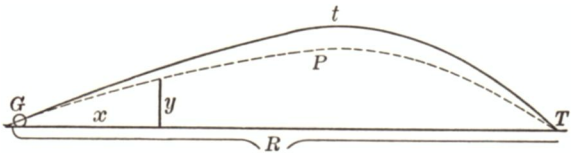
$$x = \frac{4y}{R - x}.$$

A review of literal equations grew out of this section.

This same problem of firing from behind a mask opened the subject of solution of triangles by use of trigonometric functions. The situation of the guns, the aiming point, the battery commander's station and the problems to be solved were vividly presented by diagram. The investigation led to the study of the relation of angles in a triangle, and the solution of a triangle by trigonometry.

In the diagram, T is the target and also represents the angle BTG at T. P is the aiming point, visible from all of the guns, G_1 to G_4, inclusive. The target T is visible from the Battery Commander's station B, but not from the guns which are placed

behind the mask M. The angle through which B turns his panoramic sights is read directly, and the distances BG, G_1G_2, G_2G_3, BP, etc., are known. If BT is found by using range finder or mils rule, the angle at T can be estimated in mils. Then the



azimuth through which G_1 must be revolved from direction G_1P is $\angle d + \angle a$. Call it G_1. Call the angle PBT, B. Then $B = x + y$. Since

$$x = a + T$$
$$y = d + P.$$
$$\therefore x + y = a + d + T + P$$

or

$$B = G_1 + T + P$$

and

$$G_1 = B - P - T$$
$$= B + (-P - T).$$

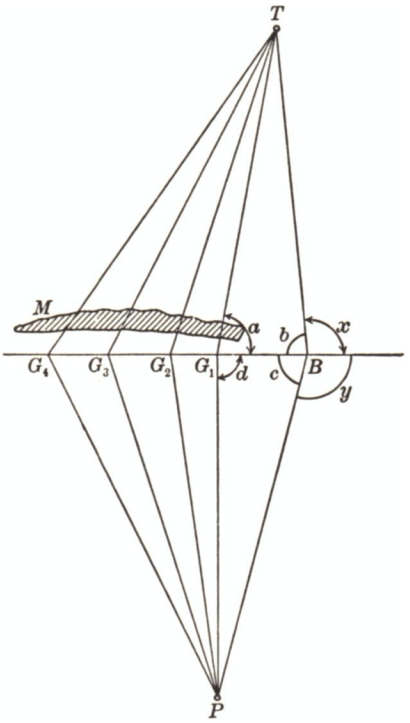
When aiming point is in front of line G_1B instead of in the rear, and to left or right of T, it is easily seen that

$$G_1 = B + (P - T).$$

Here we have a motive for studying the relations of angles in a triangle, exterior angle, etc. Further, when BG_1 is known and BT given by range finder, the class discussion very naturally led to the question "Can the range $G_{1_1}T$ be calculated?" Right here the fact that sines of angles are involved was brought out, and a lesson was given in trigonometric functions. Tables of natural functions were mastered and triangles solved, using the law of sines before new gunnery problems were considered.

In the case of the navigation course, angles and angle relations were taught in connection with the compass; charts and chart reading involved teaching some rudimentary mathematical geography, and piloting brought up the solution of triangles. Thus: In locating the ship by the one point bearing method. Present

to the class by diagram the situation. Read the log when the ship is in such position (S) that the known object L is one point forward of (or abaft) the beam, and also when it is abeam at S'. The distance $S'L$ will be 5 times the distance run. Now the question is at once brought up, Why? Of course, it is be-



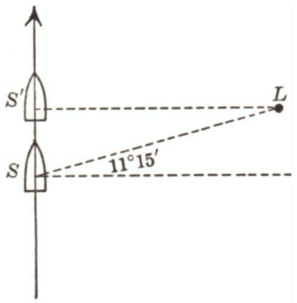
cause $S'L = SS'$ cot 11° 15′ and cot 11° 15′ is approximately 5 (5.0273). But right here in piloting a number of the methods called for the use of natural functions, and the eagerness with which the students took up and mastered them, the use of the tables, and the solution of the right triangle showed that this was the natural method of presentation. Trigonometry grew up or differentiated out of the investigation of space relations which were being studied because of a vital interest in them on the part of the students. Plane sailing and dead reckoning furnished practice for the knowledge of how to solve the right triangle trigonometrically.

To understand the use of the traverse table in Bowditch, the student needed to be able to interpret these figures, and understand that in Fig. 1

Distance $\times$ cos course angle $=$ D.Lat. or

$$\text{distance} = \frac{\text{D.Lat.}}{\cos \text{course angle}}. \qquad (1)$$

In Fig. 2, AB is the departure of a ship, R is the radius of the earth, r the radius of the parallel of latitude over which the ship



sailed (AB). DOB is the latitude of the ship, COD is the D.Long. corresponding to the departure AB.

Now

$$AB : CD = r : R$$

$$= R\,(\sin 90 - \text{Lat.}) : R.$$

That is,

$$\text{Dep.} : \text{D.Lo.} = R \cos \text{Lat.} : R.$$

$$\therefore \text{D.Lo.} = \frac{\text{Dep.}}{\cos.\ \text{Lat.}}. \qquad (2)$$

The similarity, or rather identity of form, of equations (1) and (2) explains why the same traverse table can be used in finding D.Lat. and Departure and D.Lo. It is easy to see that when the plane sailing and dead reckoning situations were presented to the student and he was shown that cosines and sines were involved, his previous experience with them in piloting would help, and give him a needed review.

Some Results.—Although half the students dropped out of the courses at the signing of the armistice, those who finished

the courses became interested in solid geometry and trigonometry and at the end of January, when new classes were organized, the class in trigonometry was three times as large as during the fall term. Ours is a commercial school, and solid geometry and trigonometry are only tolerated for those boys who "find themselves" as not adapted to commercial careers, and want to fit themselves for technical or other colleges. Those boys who took the courses in navigation or artillery and are now taking trigonometry are doing so because they see a need for mastery of trigonometry as a *science,* so that they may more thoroughly understand the *rationale* of a field in which they are interested. It is to them a means of interpreting life.

The ordinary teacher of mathematics may object that such methods of approach to high-school mathematics take too much time and require too much knowledge of applied fields on the part of the teacher. The answer is that if we would follow the line of natural evolution in the genesis of knowledge, life situations and problems in which the student is interested must arouse the desire for mathematics as an aid in solving these problems. It is the teacher's province to master the mathematics of these applications, to be able to present the subject matter of secondary-school mathematics through these problems. Commercial mathematics of an advanced type, and even simpler business calculations afford rich mines of material for such algebraic topics as literal equations, the binomial theorem, series and logarithms. How many high-school teachers know that the formula

$$A_0 = \frac{R}{i}\left\{1 - \frac{1}{(1+i)n}\right\}$$

by which the basis price of a bond may be calculated when basis rate, interest rate, time to run are given, is best derived by the formula for the *sum* in a geometric progression? Why should not an investment, installment payment, or bond problem be the avenue through which geometric progression is presented to the student? It can be done. We are doing it in the High School of Commerce.

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